

<b>Noname manuscript No.</b> (will be inserted by the editor)
--

TOMASELLI · KÜHL · URSESCU ·  
FRITZSCHE

# Correlation Effects on the Charge Radii of Exotic Nuclei

Received: date / Accepted: date

**Abstract** The structures and distributions of light nuclei are investigated within a microscopic correlation model. Two particle correlations are responsible for the scattering of model particles either to low momentum- or to high momentum-states. The low momentum states form the model space while the high momentum states are used to calculate the G-matrix. The three and higher order particle correlations do not play a role in the latter calculation especially if the correlations induced by the scattering operator are of sufficient short range. They modify however, via the long tail of the nuclear potential, the Slater determinant of the (A) particles by generating excited Slater's determinants.

**Keywords** nuclear models · distribution · charge radii

**PACS** 21.10.Ma · 21.60.-n · 21.10.Ft · 21.60.Fw

---

M. TOMASELLI

Physics Department, TUD Darmstadt, D-64289-Darmstadt, Germany  
Gesellschaft für Schwerionenforschung (GSI), D-64291 Darmstadt, Germany  
E-mail: m.tomaseli@gsi.de

T. KÜHL

Gesellschaft für Schwerionenforschung (GSI), D-64291 Darmstadt, Germany

D. URSESCU

Gesellschaft für Schwerionenforschung (GSI), D-64291 Darmstadt, Germany

S. FRITZSCHE

Institut für Physik, Kassel University, D-34132 Kassel, Germany

## 1 Introduction

Correlation effects in nuclei have first been introduced in nuclei by Villars [1], who proposed the unitary-model operator (UMO) to construct effective operators. The method was implemented by Shakin [2] for the calculation of the G-matrix from hard-core interactions.

The UMO is based on the separation of the two body potential in a short and a long components. Within this separation the effective n-body Hamiltonian contains only the long component. The short-range component is considered up to the two body correlation and produces no energy shift in the pair state.

Non perturbative approximations of the UMO have been recently applied to even nuclei in Ref. [3] which is treated here in more detail. The basics formulas of the Boson Dynamic Correlation Model (BDCM) presented in the above quoted paper have been obtained by solving the n-body problem in terms of the long range component of the two-body force. This component has the effect of generating a new correlated model space (effective space) which departs from the originally adopted one (shell model). The amplitudes of the model wave functions are calculated in terms of non linear equation of motions (EoM).

By linearizing the systems of commutator equations, which characterize the EoM, we derive the eigenvalue equations for our model space. Within this correlated formalism we generate a model that includes not only the ladder diagrams of Ref. [4] but also the folded diagrams of Kuo [5].

The n-body matrix elements which define the eigenvalue equations are calculated exactly via the Cluster Factorization Theory (CFT) [6].

In this paper the BDCM model is applied to calculate the influence of the correlations on the charge distributions of the lithium isotopes and on the charge distribution of  ${}^6\text{He}$ .

The value obtained for the charge radius of the correlated  ${}^6\text{He}$  is slightly bigger than the radius calculated in other theories [7,9] and that derived within the isotopic-shift IS theory [10]. A charge radius which agrees with the radii calculated in the Refs. [7, 9] and those calculated in the cluster models of Refs. [11-13] is on the other hand obtained by considering only two protons in the  $s_{\frac{1}{2}}$ . This non correlated radius agrees also with the radius derived at Argonne within the IS theory [10]. Correlations have therefore the property to increase the charge radius of  ${}^6\text{He}$  as observed for the isotopes of Lithium.

The calculations performed in Ref. [14] for the charge radii of the lithium isotopes, although in good agreement with those measured at GSI-TRIUMF [15] and analyzed with the help of Ref. [16], are always slightly larger than those measured. For the stable isotope  ${}^6\text{Li}$  the calculated radius agrees with the value obtained with the electron scattering experiments of Ref. [17]. However, the charge radii calculated in the IS theory could also depend on the nuclear correlations. The calculations performed in Ref. [18] for the field shift (FS) of  ${}^7\text{Li}$  show that the departure from a point nuclear approximation is a rather large effect. Additionally the higher order cross term contributions of Ref. [19] need to be considered.

A direct comparison between the calculated and the measured charge radii should therefore be performed after an accurate analysis of these two correcting factors.

## 2 Theory of Two Correlated Particles

In order to describe the structures and the distributions of nuclei we start from the following Hamiltonian:

$$H = \sum_{\alpha\beta} \langle \alpha|t|\beta \rangle a_{\alpha}^{\dagger} a_{\beta} + \sum_{\alpha\beta\gamma\delta} \langle \Phi_{\alpha\beta}|v_{12}|\Phi_{\gamma\delta} \rangle a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\delta} a_{\gamma} \quad (1)$$

where  $v_{12}$  is the singular nucleon-nucleon two body potential. Since the matrix elements  $|\alpha\beta\rangle$  are uncorrelated the matrix elements of  $v_{12}$  are infinite. This problem can be avoided by taking matrix elements of the Hamiltonian between correlated states. In this paper the effect of correlation is introduced via the  $e^{iS}$  method. In dealing with very short range correlations only the  $S_2$  part of the correlation operator need to be considered.

Following Ref. [2] we therefore calculate an “effective Hamiltonian” by using only the  $S_2$  correlation operator obtaining:

$$\begin{aligned} H_{eff} &= e^{-iS_2} H e^{iS_2} = \sum_{\alpha\beta} \langle \alpha|t|\beta \rangle a_{\alpha}^{\dagger} a_{\beta} + \sum_{\alpha\beta\gamma\delta} \langle \Psi_{\alpha\beta}|v_{12}^l|\Psi_{\gamma\delta} \rangle a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\delta} a_{\gamma} \\ &= \sum_{\alpha\beta} \langle \alpha|t|\beta \rangle a_{\alpha}^{\dagger} a_{\beta} + \sum_{\alpha\beta\gamma\delta} \langle \Psi_{\alpha\beta}|v|\Psi_{\gamma\delta} \rangle a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\delta} a_{\gamma} \end{aligned} \quad (2)$$

where  $v_{12}^l$  is the long component of the two body interaction (note that the  $v_{12}^l$  is in the following equations simply denoted as  $v$ ). The  $\Psi_{\alpha\beta}$  is the two particle correlated wave function:

$$\Psi_{\alpha\beta} = e^{iS_2} \Phi_{\alpha\beta} \quad (3)$$

In dealing with complex nuclei however the  $(S_i, i = 3 \cdots n)$  correlations should also be considered.

The evaluation of these diagrams is, due to the exponentially increasing number of terms, difficult in a perturbation theory.

We note however that one way to overcome this problem is to work with  $e^{i(S_1+S_2+S_3+\cdots+S_i)}$  operator on the Slater’s determinant by keeping the n-body Hamiltonian unvaried.

After having performed the diagonalization of the n-body Hamilton’s operator we can calculate the form of the effective Hamiltonian which, by now, includes correlation operators of complex order.

Using Eq. (1), we can compute the commutator of the Hamiltonian with the operator  $(a_{j_1}^{\dagger} a_{j_2}^{\dagger})^J$  that creates a valence particle pair. By performing this calculation we shall retain the linear and the non-linear terms which are formed by coupling the shell model states with the particle-hole excitations of the core.

At this point in order to obtain a complete system of equations, we also have to calculate the commutator of the Hamiltonian (1) with the non linear terms. With this calculation we introduce in the model space states which are

formed by coupling the valence states with the two particle-hole excitations of the core.

The successive model equations are then formed by calculating the commutator with the operator obtained in the previous step.

The set of commutator equations above is suitable to be solved by means of a perturbation expansion. The perturbative solution of the system of commutator equations is however not easily obtainable due to the high number of diagrams one needs to calculate.

A non perturbative solution of the system of commutator equation can be obtained within the linearization method, which consists by applying the Wick's theorem to the  $((n+2)p-2h)$  terms and by neglecting the normal order terms.

This approximation is motivated by the consideration that the low lying spectra of nuclei the  $((n+2)p-2h)$  terms are lying at much higher energy than that of the  $((n+1)p-1h)$  states.

The linearized system of the commutator equations is then solved exactly in terms of the CFT which calculates the n-body matrix elements in an expedite and exact way.

In the following we give the basic formula of the method for a (nuclear) system with two valence particles. In second quantization, the two particle states are defined by:

$$\Phi_{2p} \longrightarrow \Phi_{j_1 j_2}^J = A_1^\dagger(\alpha_1 J)|0\rangle = [a_{j_1}^\dagger a_{j_2}^\dagger]^J_M |0\rangle, \quad (4)$$

where, for the sake of simplicity, we have omitted the isospin quantum numbers and where

$$\alpha_1 \leftrightarrow j_1 j_2 \quad (5)$$

has been introduced to ensure a compact index notation of the angular momenta of the two particles. In this notation, the operator product  $a_{j_1}^\dagger a_{j_2}^\dagger$  just creates two *coupled* particles of single particle  $j_1$  and  $j_2$  *coupled* to the final J quantum number.

To derive the effect of the correlation on the two valence particles, we have, at this stage, to evaluate the next commutator

$$\begin{aligned} & [H, A_1^\dagger(\alpha_1 J)]|0\rangle \\ &= [(\sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^\dagger a_{\alpha} + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|v(r)|\gamma\delta\rangle a_{\alpha}^\dagger a_{\beta}^\dagger a_{\delta} a_{\gamma}), (a_{j_1}^\dagger a_{j_2}^\dagger)^J]|0\rangle \end{aligned} \quad (6)$$

which, after some operator algebra becomes

$$\begin{aligned} & [H, A_1^\dagger(\alpha_1 J)]|0\rangle \\ &= \sum_{\beta_1} \Omega(2p|2p') A_1^\dagger(\beta_1 J)|0\rangle + \sum_{\beta_2 J'_1 J'_2} \Omega(2p|3p1h) A_2^\dagger(\beta_2 J'_1 J'_2)|0\rangle. \end{aligned} \quad (7)$$

In Eq. (7) the  $A_1^\dagger(\beta_1 J)$  operators are those of Eq. (4) and the  $A_2^\dagger(\beta_2 J'_1 J'_2 J)$  are defined below:

$$\Phi_{3p1h} \longrightarrow \Phi_{j'_1 j'_2 j'_3 j'_4}^{J'_1 J'_2 J} = A_2^\dagger(\beta_2 J'_1 J'_2 J)|0\rangle = ((a_{j'_1}^\dagger a_{j'_2}^\dagger)^{J'_1} (a_{j'_3}^\dagger a_{j'_4}^\dagger)^{J'_2})^J |0\rangle. \quad (8)$$

In Eq. (8) we have used the additional convention:

$$\beta_2 \longrightarrow j'_1 j'_2 j'_3 j'_4 \quad (9)$$

and we have associated:

$$\begin{aligned} J'_1 &\text{ to the coupling of } j'_1 j'_2 \\ J'_2 &\text{ to the coupling of } j'_3 j'_4 \end{aligned} \quad (10)$$

Having extended the commutator as in Eq. (7), we have also to calculate the commutator equation for the  $A_2^\dagger(\alpha_2 J_1 J_2 J)$  operators as given below:

$$\begin{aligned} &[H, A_2^\dagger(\alpha_2 J_1 J_2 J)]|0\rangle \\ &= \sum_{\beta_2 J'_1 J'_2} \Omega(3p1h|3p'1h') A_2^\dagger(\beta_2 J'_1 J'_2 J)|0\rangle + \sum_{\beta_3 J'_1 J'_2 J'_3} \Omega(3p1h|4p2h) A_3^\dagger(\beta_3 J'_1 J'_2 J'_3 J)|0\rangle, \end{aligned} \quad (11)$$

where we have introduced the (4p-2h) wave functions defined below:

$$\begin{aligned} \Phi_{4p2h} &\longrightarrow \Phi_{j'_1 j'_2 j'_3 j'_4 j'_5 j'_6}^{J'_1 J'_2 J'_3 J'_4 J'_5 J'_6} = A_3^\dagger(\beta_3 J'_1 J'_2 J'_3 J'_4 J'_5 J'_6)|0\rangle \\ &= (((a_{j'_1}^\dagger a_{j'_2}^\dagger)^{J'_1} (a_{j'_3}^\dagger a_{j'_4}^\dagger)^{J'_2})^{J'_3} (a_{j'_5}^\dagger a_{j'_6}^\dagger)^{J'_4})^{J'_5} |0\rangle, \end{aligned} \quad (12)$$

and where we have consistently extended the definition given in (5,10):

$$\beta_3 \longrightarrow j'_1 j'_2 j'_3 j'_4 j'_5 j'_6 \quad (13)$$

with:

$$\begin{aligned} J'_1 &\text{ associated to the coupling of } j'_1 j'_2 \\ J'_2 &\text{ associated to the coupling of } j'_3 j'_4 \\ J'_3 &\text{ associated to the coupling of } j'_5 j'_6 \end{aligned} \quad (14)$$

In the definition of  $A_3^\dagger(\beta_3 J'_1 J'_2 J'_3 J)$  the coupling of  $J'_1$  to  $J'_2$  to  $J'_3$  has been discarded from the notation. In Eqs. (7,11) the  $\Omega$ 's are the matrix elements of the Hamilton's operator in the model wave functions. The next step would then be the computation of the commutator of the Hamiltonian with the  $A_3^\dagger(\beta_3 J'_1 J'_2 J'_3 J)$  operators. Here we linearize these contributions by considering that in the study of the low energy spectrum and in the calculation of ground-state correlated distributions the  $A_3^\dagger(\beta_3 J'_1 J'_2 J'_3 J)$  terms have a small contribution. The linearization is performed by applying to the (4p2h) terms:

$$\sum_{\alpha\beta\gamma\delta} \langle \alpha\beta|v(r)|\gamma\delta\rangle a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma A_3^\dagger(\beta_3 J'_1 J'_2 J'_3 J) \quad (15)$$

the Wick's theorem and to discard the normal order terms. Within this linearization approximation we generate non perturbative solutions of the EoM from the commutator equations of Eq. (7,11), i.e.: the eigenvalue equations for the mixed mode system:

$$\begin{aligned} [H, A_1^\dagger(\alpha_1 J)]|0\rangle &= \sum_{\beta_1} \Omega(2p|2p') A_1^\dagger(\beta_1 J)|0\rangle \\ &+ \sum_{\beta_2 J'_1 J'_2} \Omega(2p|3p'1h') A_2^\dagger(\beta_2 J'_1 J'_2 J)|0\rangle, \end{aligned} \quad (16)$$

and

$$\begin{aligned} [H, A_2^\dagger(\alpha_2 J_1 J_2 J)]|0\rangle &= \sum_{\beta_1} \Omega(3p1h|2p') A_1^\dagger(\beta_1 J)|0\rangle \\ &+ \sum_{\beta_2 J'_1 J'_2} \Omega(3p1h|3p'1h') A_2^\dagger(\beta_2 J'_1 J'_2 J)|0\rangle. \end{aligned} \quad (17)$$

Within the application of the GLA approximation we convert Eqs. (7, 11) in an eigenvalue equation for the configuration mixing wave functions (CMWFs) of the model. In fact, the linearization provides the additional matrix elements necessary to write the following identity:

$$\Omega(3p1h|3p'1h') = \langle j_1 j_2 j_3 j_4 | v(r) | j'_1 j'_2 j'_3 j'_4 \rangle, \quad (18)$$

and to introduce the off-diagonal matrix elements which couple the (2p) to the (3p1h) subspaces. Now, by writing Eqs. (16,17) in the following matrix form:

$$\begin{pmatrix} [H, A_1^\dagger(\alpha_1 J)]|0\rangle \\ [H, A_2^\dagger(\alpha_2 J_1 J_2 J)]|0\rangle \end{pmatrix} = \begin{pmatrix} E_{2p} + \Omega(2p|2p') & \Omega(2p|3p'1h') \\ \Omega(3p1h|2p') & E_{3p1h} + \Omega(3p1h|3p'1h') \end{pmatrix} \begin{pmatrix} A_1^\dagger(\beta_1 J)|0\rangle \\ A_2^\dagger(\beta_2 J_1 J_2 J)|0\rangle \end{pmatrix}, \quad (19)$$

and by multiplying to the left with:

$$\begin{pmatrix} \langle 0 | A_1(\alpha_1 J) \\ \langle 0 | A_2(\alpha_2 J_1 J_2 J) \end{pmatrix} \quad (20)$$

we generate the eigenvalue equation for the dressed particles:

$$\begin{aligned} \sum_{\beta_1 \beta_2 J'_1 J'_2} &\begin{pmatrix} E_{2p} + \langle A_1(\alpha_1 J) | v(r) | A_1^\dagger(\beta_1 J) \rangle & \langle A_1(\alpha_1 J) | v(r) | A_2^\dagger(\beta_2 J'_1 J'_2 J) \rangle \\ \langle A_2(\alpha_2 J_1 J_2 J) | v(r) | A_1^\dagger(\beta_1 J) \rangle & E_{3p1h} + \langle A_2(\alpha_2 J_1 J_2 J) | v(r) | A_2^\dagger(\beta_2 J'_1 J'_2 J) \rangle \end{pmatrix} \\ &\cdot \begin{pmatrix} \chi_1(\beta_1 J) \\ \chi_2(\beta_2 J'_1 J'_2 J) \end{pmatrix} = E \begin{pmatrix} \chi_1(\alpha_1 J) \\ \chi_2(\alpha_2 J_1 J_2 J) \end{pmatrix} |0\rangle. \end{aligned} \quad (21)$$

In Eq.(21)  $E_{2p} = \epsilon_{j_1}^{HF} + \epsilon_{j_2}^{HF}$  and  $E_{3p1h} = \epsilon_{j_1}^{HF} + \epsilon_{j_2}^{HF} + \epsilon_{j_3}^{HF} - \epsilon_{j_4}^{HF}$  are the Hartre-Fock energies while the  $\chi$ 's are the projections of the model states:

$$|\Phi_{2p}^J\rangle = \chi_1(\alpha_1 J) A_1(\alpha_1 J)|0\rangle + \chi_2(\alpha_2 J_1 J_2 J) A_2(\alpha_2 J_1 J_2 J)|0\rangle \quad (22)$$

to the basic vectors 2p, 3p1h. To conclude, although the (4p-2h) CMWFs are not active part of the model space, they are important for structure calculations. One may therefore associate the GLA approximation to a parameter which describes the degree of complexity of the model CMWFs. Within the first order linearization we obtain the EoM for the shell model while within the second and third order linearization approximations we derive the EoM of valence particles coexisting with the complex particle-hole structure of the excited states.

In this paper we solve Eq. (21) self-consistently. The solutions for the first iteration step are obtained by diagonalizing the eigenvalue equation (21). The first step of the iterative method generates the dynamic amplitudes for the two dressed particles, i.e. two particles coexisting with the 3p1h structures. With the calculated eigenvectors we recompute then the matrix elements  $\langle j_1 j_2 | v(r) | j_1 j_2 j_3 j_4 \rangle$  and  $\langle j_1 j_2 j_3 j_4 | v(r) | j'_1 j'_2 j'_3 j'_4 \rangle$  and we diagonalize again the eigenvalue equation. The iterations are repeated until the stabilization of the energies has been reached. Before performing the diagonalization of relative Hamilton's operator in the CMWFs base we have to eliminate the spurious center of mass components. In the BDCM this is performed, following the calculations of Refs. [20, 21], by calculating the percent weights of spurious states in the model wave functions. These can be obtained by evaluating the energy of the center of mass according to the following equation:

$$E_R = \int dR \Psi^{\dagger \text{dressed}}(j_i j_j J) (R2) \Psi^{\text{dressed}}(j_i j_j J) + 2 \sum_{ij} \int d\mathbf{r}_i d\mathbf{r}_j \Psi^{\dagger \text{dressed}}(j_i j_j J) (\mathbf{r}_i \cdot \mathbf{r}_j) \Psi^{\text{dressed}}(j'_i j'_j J). \quad (23)$$

In Eq. (23) the calculation of the integrals can be performed by using the CFT expansion for the (3p1h) states and by considering that for two particle states we have:

$$\langle j_i j_j J | (\mathbf{r}_i \cdot \mathbf{r}_j) | j_i j_j J \rangle = \frac{4\pi}{3} [\hat{j}_i \hat{j}_j] \begin{pmatrix} j_i & 1 & j_j \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} 2 \left\{ \begin{matrix} j_i & j_j & J \\ j_i & j_j & 1 \end{matrix} \right\} \langle l_i | r | l_j \rangle 2, \quad (24)$$

where:

$$\hat{j} = (2j + 1). \quad (25)$$

By diagonalizing the above operator in the model space we obtain the energy of the center of mass. The overlap with the model space give the degree of “spuriousity” of the different components. The model space which characterize the BDCM is formed by adding even particle to a closed-shell nucleus. The closed shell configuration can be described by a single Slater determinant and one can use the Hartree-Fock's theory to obtain the binding energy and the single-particle energies. Alternatively one can remark that for a closed shell nucleus (Z,N) the single particle energies for the states above the Fermi surface are related to the binding energies differences:

$$\epsilon_p^> = BE(Z, N) - BE^*(Z + 1, N), \quad (26)$$

**Table 1** Single-particle scheme and single particle energies (MeV) used to form the model CMWFs for the A=6 isotopes

hole	$1s_{1/2}$							
energy	-20.58							
hole/particle	$1p_{3/2}$							
energy	1.43							
particle	$1p_{1/2}$	$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}$	$1f_{7/2}$	$2p_{3/2}$	$1f_{5/2}$	$2p_{1/2}$
energy	1.73	17.21	22.23	23.69	25.23	27.18	28.33	29.67

and

$$\epsilon_n^> = BE(Z, N) - BE^*(Z, N + 1). \quad (27)$$

The single particle energies for the states below the Fermi surface are given by:

$$\epsilon_p^< = BE^*(Z - 1, N) - BE(Z, N), \quad (28)$$

and

$$\epsilon_n^< = BE^*(Z, N - 1) - BE(Z, N). \quad (29)$$

The BE are ground states binding energies which are taken as positive values, and  $\epsilon$  will be negative for bound states. ( $BE^* = BE - E_x$ ) is the ground state binding energy minus the excitation energy of the excited states associated with the single particle states. Within this method, which recently has been reintroduced by B.A. Brown [22], we derive the single particle energies from the known spectra of neighbor nuclei (see Table (1)).

The generalization of the previously defined formalism needed to calculate the charge radii of the A=7 (one particle DCM), A=8 (four particles BDCM) and A=11 (three particles DCM) is not given explicitly in this paper, but will be presented shortly.

## 2.1 Results

In order to perform structure calculations, we have to define a single particle base with the relative single-particle energies and to choose the nuclear two-body interactions. The single-particle energies of these levels are taken from the known experimental level spectra of the neighboring nuclei and given in Table (1). For the experimentally unknown single particle energies of the fp shells we use the corresponding energies for the mass A=9 nuclei scaled accordingly the different binding energies. In this paper we perform as in Ref. [9] calculations by assuming all levels as bound for the particle-particle interaction, we use the G-matrix obtained from Yale potential [23]. These matrix elements are evaluated by applying the  $e^S$  correlation operator, truncated at the second order term of the expansion, to the harmonic oscillator base with size parameter  $b=1.76$  fm. As also explained in Ref. [3] the potential used by the BDCM is separated in low and high momentum components. Therefore, the effective model matrix elements calculated within the present separation method and those calculated by Kuo [24] are pretty similar. The separation method generates matrix elements, which are almost independent from the radial shape of the different potentials generally used in structure



**Table 2** Calculated charge radii for  ${}^6\text{He}$  in fm compared with the results obtained in other theoretical models and with the radius derived within the IS theory.

charge radius of ${}^6\text{He}$	Model
1.944 [7]	no-core shell model
2.09 [9]	quantum Monte Carlo technique
2.25 this work	BDCM
2.39 this work	BDCM without the folded diagrams
2.06 this work	two correlated $1s_{\frac{1}{2}}$ -protons
1.99 [11]	Cluster
1.99 [12]	Cluster
1.99 [13]	Cluster
$2.054 \pm .014$ [10]	Isotopic Shift (Exp.)

calculations.

The particle-hole matrix elements could be calculated from the particle-particle matrix elements via a re-coupling transformation. We prefer to use the phenomenological potential of Ref. [25]. The same size parameter as for the particle-particle matrix elements has been used. In Table (2) the calculated charge radii of  ${}^6\text{He}$  are compared with the radii calculated by the other theoretical models and with the radius obtained by the IS theory. In the theoretical models quoted in this table the calculations of the charge distributions and of the charge radii are performed in terms of non- correlated operators. The correlations are included only in the derivation of the  $S_2$  effective Hamiltonian.

For the stable  ${}^6\text{Li}$  the calculated charge radius is equal to 2.55 fm, a value that reproduces well the charge radius of 2.55 fm obtained in Ref. [17] from the electron scattering experiments. The charge radii given in Table (3) for other lithium isotopes are however larger then those calculated in the other quoted theoretical models and then those obtained within the IS theory. Here also the main difference between the results obtained in the DCM and BDCM models and those of the other theoretical calculations has to be found in the treatment of the correlation operator. In Ref. [7] the charge radii are calculated in the “no core shell model” which is based on exact solutions of the two particles Schrödinger’s equation by considering large computational spaces. The calculations do not include however the  $S_3$  correlations. Ref. [9] presents charge radii evaluated within an accurate Quantum Monte Carlo Method. The Hamiltonian used includes two- and three-bodies forces in a two-body correlated mechanics. The calculation method of Ref. [27] is based on a microscopic cluster method in which few particles are interacting with the rest nucleus considered in its ground state. In our model the excitations of the core are associated to the  $S_3$  correlation operator which increases the charge radius of the Lithium isotopes.

### 3 Conclusions and Outlook

In this contribution we have investigated the effect of the microscopic correlation operators on the charge distributions of  ${}^6\text{He}$  and of the Lithium isotopes.

**Table 3** Calculated charge radii for the Lithium isotopes in fm compared with the results obtained in other theoretical models and with the radius derived within the IS theory (Exp.).

Lithium	Exp.(GSI) [15]	Exp.+Theo. [26]	Theo. [7]	Theo. [9]	Theo. [27]	DCM+BDCM
	rms	rms	rms	rms	rms	
<sup>6</sup> Li	2.55	2.47	2.22	2.54	-	2.55
<sup>7</sup> Li	2.46	2.43	2.13	2.41	2.43	2.41
<sup>8</sup> Li	2.37	2.42	2.13	2.26	2.34	2.40
<sup>9</sup> Li	2.30	2.34	2.16	2.21	2.27	2.42
<sup>11</sup> Li	2.47	3.01	-	-	2.57	2.67

The microscopic correlation has been separated in short- and long-range correlations according the definition of Shakin [2]. The short-range correlation has been used to define the effective Hamiltonian of the model while the long-range is used to calculate the structures and the distributions of exotic nuclei. As given in the work of Shakin, only the two-body short-range correlation need to be considered in order to derive the effective Hamiltonian especially if the correlation is of very short range. For the long range correlation operator the three body component is important and should not be neglected. Within the three body correlation operator one introduces in the theory a three body interaction which compensates for the use of the genuine three body interaction of the no-core shell model.

By using generalized linearization approximations and cluster factorization coefficients we can perform expedite and exact calculations.

Within the calculated correlated distributions we obtain charge radii slightly larger than those calculated for non correlated distributions and derived by the IS experiments.

The application of the DCM [18] to the two and three electron energies and distributions of the Helium and Lithium atoms respectively could serve as future motivation for a reevaluation of the IS theory. From one side, the calculation of CM of the different isotopes, could help to obtain a non-perturbative formulation of the isotopic change of the electron transition energies. From the other side the field shift theory could include the correct isotopic variation. Since the derivation of the charge radii from the two photon experiments is influenced by the precision of the theoretical calculations, this new proposed method could contribute to evaluate with better precision the charge radii of exotic nuclei.

## References

1. Villars F., *Proc. Enrico Fermi Int. School of Physics XXII*, Academic Press N.Y. (1961).
2. Shakin C. M. and Waghmare Y. R., *Phys. Rev. Lett.* **16** (1966), 403; Shakin C. M., Waghmare Y. R. and Hull M. H., *Phys. Rev.* **161** (1967), 1006.
3. Tomaselli M., Liu L. C., Fritzsche S., Kühl T. and Ursescu D., *Nucl. Phys. A738* (2004), 216; Tomaselli M., Liu L. C., Fritzsche S. and Kühl T. *J. Phys. G: Nucl. Part. Phys.* **30** (2004), 999.
4. Brückner K.A., *The Many Body Problem*, John Wiley and Sons, New York (1959).

- 
5. Kuo T. T. S. and Osnes E., *Folded-Diagrams Theory of the Effective Interaction in Atomic Nuclei*, Springer Lecture Notes in Physics Vol. 366, Berlin (1991).
  6. Tomaselli M., Kühl T., Ursescu D. and Fritzsche S., *Prog. Theor. Phys.* **116** (2006), 699.
  7. Navrátil P. and Barrett B. R., *Phys. Rev.* **C57** (1998), 3119; Navrátil P. and Caurier E., *Phys. Rev.* **C69** (2004), 014331.
  8. Stancu I., Barrett B. R., Navrátil P. and Vary J. P., *Phys. Rev.* **C71** (2005), 044325.
  9. Pieper S. C. and Wiringa R. B., *Ann. Rev. Part. Sci.* **51** (2001), 53; *Phys. Rev.* **C66** (2002), 044310.
  10. Wang L.-B., Müller P., Bailey V. *et al.*, *Phys. Rev. Lett.* **93** (2004), 142501.
  11. Würzer J. and Hofmann H. M., *Phys. Rev.* **C55** (1997), 688.
  12. Funada S., Kaneyama H. and Sakuragi Y., *Nucl. Phys.* **A575** (1994), 93.
  13. Varga K., Suzuki Y. and Ohbayasi Y., *Phys. Rev.* **C50** (1994), 189.
  14. Tomaselli M., Kühl T., Nörtershäuser W., Ewald G., Sanchez R., Fritzsche S., and Karshenboim G. S., *Can. J. Phys.* **80** (2002), 1347.
  15. Ewald G., Nörtershäuser W., Dax A., *et al.*, *Phys. Rev. Lett.* **93** (2004), 113002; Bushaw B. A., Nörtershäuser W., Ewald G. *et al.*, *Phys. Rev. Lett.* **91** (2003), 043004; Sánchez R., Nörtershäuser W., Ewald G., Albers D., Behr J., Bricault P., Bushaw B. A., Dax A., Dilling J., Dombsky M., Drake G. W. F., Götze S., Kirchner R., Kluge H.-J., Kühl T., Lassen J., Levy C. D. P., Pearson M. R., Prime E. J., Ryjov V., Wojtaszek A., Yan Z.-C., Zimmerman C. *Phys. Rev. Lett.* **96** (2005), 033002.
  16. Yan Z.-C. and Drake G. W. F. *Phys. Rev.* **A66** (2002), 042504.
  17. Li G. C., Sick I., Whitney R. R. and Yearian M. R., *Nucl. Phys.* **A162** (1971), 583.
  18. Tomaselli M., *Can. J. Phys.* **83** (2005), 467.
  19. Aufmuth P., *J. Phys. B: At. Mol. Phys.* **15** (1982), 3127.
  20. Baranger E. and Lee C. W., *Nucl. Phys.* **22** (1961), 157.
  21. Unna I. and Talmi I., *Phys. Rev.* **112** (1958), 452.
  22. Brown B. A., *Prog. Part. Nucl. Phys.* **47** (2001), 524.
  23. Shakin C. M., Waghmare Y. R., Tomaselli M. and Hull M. H., *Phys. Rev.* **161** (1967), 1015.
  24. Jiang M. F., Machleidt R., Stout D. B. and Kuo T. T. S., *Phys. Rev.* **C46** (1992), 910; Lacombe M., Loiseau B., Richard J. M., Vinh Mau R., Côté J., Pirès P. and de Tourreil R., *Phys. Rev.* **C21** (1980) 861; Machleidt R., *Adv. Nucl. Phys.* **19** (1989), 189; Bogner S., Kuo T. T. S., Coraggio L., Covello A. and Itaco N., *Phys. Rev.* **C65** (2002), 051301(R).
  25. Millener D.J. and Kurath D., *Nucl. Phys.* **255** (1975), 315.
  26. Tanihata I., *Phys. Lett.* **B206** (1988), 592.
  27. Suzuki Y., *Progr. Theo. Phys. Supp.* **146** (2002), 413; Varga K., Suzuki Y. and Taniata I., *Phys. Rev.* **C52** (1995), 3013; Varga K., Suzuki Y. and Lovas R. G., *Phys. Rev.* **C66** (2002), 041302.